Complex dynamics of magnetic domain walls

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Abstract

We resume the recent theoretical and experimental results which point towards a deeper comprehension of the complex dynamics of magnetic domain walls, i.e., the Barkhausen noise. In particular, we show how the theoretical framework of depinning transition is able to correctly describe the various experimental scaling exponents, included the power spectral exponent which has been investigated without success since the earlier papers.

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1. Introduction

The occurrence of many natural systems displaying complex dynamics has stimulated an intense activity in the recent past. In particular, the scale-free character of dynamical response to external perturbations has puzzled for long time both theoretical and experimental researchers. Some dynamical systems show in fact universal scaling laws and 1/f-type noise those origin is often not completely understood: for instance, this is the case of many systems where the dynamics takes place in form of avalanches, such for earthquakes, microfractures, flux lines in type-II superconductors, etc. In this respect, a unique example is given by the dynamics of magnetic domain walls in soft magnetic materials, known as Barkhausen effect, where the relative easiness in performing experiments and thus in obtaining large statistics has stimulated a large number of theoretical and experimental papers. The clicking Barkhausen noise due to the motion of domain walls has been first measured in 1909 [1], but a reasonable description of its properties
still represents an intriguing task, requiring extended methods of statistical mechanics. In last years, the most important step has been the change from a pure phenomenological description of the wall dynamics \[2,3\] to a more detailed microscopical evaluation of the relevant interactions setting the wall motion. This led to a series of models those general characteristic is the presence of an underlying critical point, responsible for the scale invariance. On one side, some authors recognized this criticality as due to the depinning transition occurring when the applied field overcomes a critical depinning field, while others considered the vicinity of a first order critical transition driven by the disorder in the frame of non-equilibrium random field Ising model (RFIM). These different approaches generated a number of theoretical predictions, especially for the values of critical exponents, directly comparable to experimental results. In this paper, we want to review and elucidate all these aspects, in order to clarify the reliability of different approaches to explain the experimental data. We first resume the models based on the depinning transition of the domain wall, then considering approaches based on RFIM, adding final comments and addressing open questions.

2. Depinning transition of a domain wall

Roughly speaking, a domain wall (of the 180° type) is the transition region between two magnetic domains of opposite magnetization. Its width depends on the competition between the exchange interaction which tends to get the spins parallel and the anisotropy field which forces the spins to align along the anisotropy direction as given, for instance, by the interaction with the lattice. Because of the intrinsic complexity of the structure given the large number of degree of freedom, it is reasonably to assume, although quite drastically, that the wall can be described by a rough surface, i.e., the domain wall width is reduced to zero. With this approximation, the effects of various interactions have to be calculated on each point of the wall. At the same time, we ignore any possible effect of the temperature. Within these hypothesis, the dynamics of a single domain wall, described by its position \(h(\vec{r},t)\), is set by the most relevant interaction fields acting on it \[4,5\]

\[
\Gamma \frac{\partial h(\vec{r},t)}{\partial t} = H - k \int d^2r' h(\vec{r}',t) + \gamma_w \nabla^2 h(\vec{r},t)
+ \int d^2r' J_{dip}(\vec{r} - \vec{r}') (h(\vec{r}') - h(\vec{r})) + \eta(\vec{r},h) .
\] (1)

This is a typical equation of a viscous motion, as the velocity of \(\partial h(\vec{r},t)/\partial t\) of a point \(\vec{r}\) is proportional to a net field, with \(\Gamma\) the viscosity constant. This field is the difference between an homogeneous external driving field \(H\) and: (i) a demagnetizing field, with \(k\) the demagnetizing factor, proportional to the magnetization \(\int d^2r' h(\vec{r}',t)\); (ii) an elastic field, with \(\gamma_w\) the surface tension; (iii) a dipolar field arising from the stray fields consequence of the wall curvature, with \(J_{dip}\) in Fourier space given by

\[
J_{dip} \sim \frac{\mu_0 M^2_s}{\sqrt{Q}} \frac{p^2}{\sqrt{p^2 + Qq^2}}
\] (2)
Table 1
Critical exponents calculated using renormalization group [12] (valid to order $\varepsilon = 2\mu - d$) and by the front propagation model of RFIM [11], compared to experimental data on polycrystalline and amorphous alloys [7]. The exponents are defined as: $P(s) \sim s^{-\tau}f(s/s_0)$, $P(T) \sim T^{-\alpha}g(T/T_0)$ for the size and duration distribution, $s_0 \sim k^{-1/\sigma_k}$ and $T_0 \sim k^{-\Delta_k}$ for the cutoffs, and $\langle s(T) \rangle \sim T^{1/\nu\zeta}$ for the relation between the average size $\langle s(T) \rangle$ of an avalanche having the duration $T$.

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<th>Depinning transition model</th>
<th>RFIM</th>
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as calculated in Ref. [6] in the case of finite anisotropy $K$, with $Q = 1 + 2\mu_0M_s^2/K$, (with $p$ and $q$ the wave vectors along $x$ and $y$, respectively). Finally, (iv) a Gaussian uncorrelated random field $\eta(\vec{r},h)$ taking into account all the possible effects of dislocations, residual stress and non-magnetic inclusions.

The elastic and dipolar fields have very different ranges, as the former is short range $J_{el} \sim \gamma_{el}(p^2 + q^2)$, while the latter is long range, as given by Eq. (2). It is worth considering the case $Q \gg 1$, valid for systems with low anisotropy (low $K$), as for instance, amorphous alloys where the lack of any crystal lattice and the random directions of local anisotropies cancel out the macroscopic anisotropy. Considering for simplicity the same wave vectors in both directions ($p \approx q$), we have $J_{el} \sim 2\gamma_{el}p^2$ and $J_{dip} \sim \mu_0M_s^2/Qp \sim 1/2Kp$, so that the dipolar term vanishes with a vanishing anisotropy. In addition, the two fields are equal for a typical length $l_c = 1/p_c = 4\gamma_{el}/K = 4\delta$, i.e., a few domain wall widths $\delta$. In other words, the short-range term prevails within distances of the order of $\delta$, while the long range one prevails at larger distances. These considerations can help to understand the differences between the experimental data of polycrystalline alloys (with quite high $K$) and amorphous systems (with vanishing anisotropy). These systems are found to belong to two distinct universality classes, showing different critical exponents of size ($s$) and duration ($T$) distributions of Barkhausen avalanches [7], and also of power spectrum [8], as reported in Table 1. The presence of grains in polycrystalline alloys keeps the anisotropy values high enough to make the long-range interaction prevail, while short-range interaction holds for amorphous systems. An interesting case, confirming the above scenario, is given by a partially crystallized Fe$_{21}$Co$_{64}$B$_{15}$, where $\alpha$-Fe crystals of about 50 nm are induced in the amorphous matrix by proper annealing. In this alloy, the domain wall width is of the order of 10 nm, and the material turned out to belong to long-range universality class, while the as-cast material belongs to the short one [7].

In general terms, considering an interaction of the type $J \sim p^\mu$, with $\mu = 1$ and $\mu = 2$ for long- and short-range interaction, respectively, the upper critical dimension results to be $d_c = 2\mu$, and critical exponents can be calculated using renormalization group calculation, as reported in Table 1. It is worth noting that not only the size and duration
critical exponents well agree with the predictions, but also the exponents relating the distribution cutoffs $s_0$ and $T_0$ to the demagnetizing factor $k$ (as we predicted in [7]), together with the exponent relating the average size $\langle s \rangle$ to the duration which turns out to be the exponent of the power spectrum [8,9].

3. Discussion

As sketched in the Introduction, a different interpretation of criticality has been suggested in the frame of the non-equilibrium zero temperature RFIM [10]. In particular, two variants has been proposed to describe the Barkhausen data [11]: (i) a single front propagation dynamics where mainly the spins at the edge of an existing front are flipping, while the others have a low probability to flip because the presence of a long-range demagnetizing field; (ii) the nucleation model, where the spins are flipping everywhere in the system. While the latter model gives critical exponents quite different from our experimental data, the former shows strong analogies with the short-range model explained above. Not only the values of size distribution exponent $\tau$ are quite close, but we found that also the size cutoff $s_0$ follows exactly the same dependence on the demagnetizing factor $k$ predicted in Ref. [7] in the contest of depinning transition, as clearly shown in Fig. 1. In fact, data at different demagnetizing factors rescale with $1/\sigma_{k} \sim 0.7$, close to the experimental value of $0.79 \pm 0.10$ found on an amorphous alloy under tensile stress [7]. We argue that the two model should belong to the same universality class, also considering the short-range nature of the interaction of these models.

![Fig. 1. Data collapse of avalanche size distribution $P(s) \sim s^{-\tau} f(s/s_0)$ simulated by the front propagation model of RFIM (see text), using different values of the demagnetizing factor $k$. The critical exponent $\tau$ is given by 1.28, while $s_0 \sim k^{-1/\sigma_k}$ with $1/\sigma_k \sim 0.7$.](image)
The other aspect that puzzled for long time many researchers in this field is a reasonable description of the power spectral density of the Barkhausen signal, which roughly has a $f^{-\beta}$ dependence at high frequencies and a more complicate pattern a low frequencies [8]. We recently found [8] indications that the two universality classes could hold also in this case, as we found that $\beta \sim 1/\sigma v z$ (with $\sigma, v, z$ are three other critical exponents) as predicted in [9], with different values 1.77 and 2 for the two classes (see Table 1). In any case, this general behavior has to be confirmed in the case of more samples and applying different external field driving rates, as this changes significantly the power spectrum shape, or, in other words, the correlation within and between avalanches. In this respect, the recent approach introduced in [9,11] to consider the pulse shape scaling as the key ingredient to calculate the spectral properties appears very useful and promising, and has to be applied for extended statistical analysis of spectral properties.

References