Signature of negative domain wall mass in soft magnetic materials

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Abstract

Magnetic properties of ferromagnetic materials often originate from domain wall motion, involving different damping mechanisms, an effective mass and various interactions with the surrounding media. In metallic materials, eddy current damping overwhelms inertia and thus the effect of the mass is usually neglected. We have recently reported experimental evidence that in soft metallic ferromagnets eddy currents yield an observable negative contribution to the effective domain wall mass. The weight of this mass is of the order of $10^{-5}$ kg/m\textsuperscript{2}, much larger than the positive Doring mass ($\sim10^{-9}$ kg/m\textsuperscript{2}). This negative effective mass is responsible for the leftward asymmetry of Barkhausen noise pulse shapes. In particular, this asymmetry depends on the pulse duration and it is found to encode important information on the characteristic time of the underlying domain wall dynamics. Only on long timescales the pulse shapes are symmetric and show the universal features typical of the Barkhausen effect. This result clarifies the general significance of pulse shape asymmetry commonly observed in systems showing a similar crackling noise, and contributes to better understand the microscopic phenomena responsible of magnetic hysteresis.

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1. Introduction

The dynamics of domain walls under the action of a magnetic field is responsible for most magnetic properties, such as hysteresis, coercivity, and magnetization noise, as widely studied in the literature. Domain walls can also exhibit an effective mass [1,2], which is connected to the increase of the wall energy with the velocity, as experimentally revealed, for instance, in the dynamic susceptibility of insulating ferrite materials [3]. Most ferromagnetic materials are, however, conducting and inertial effects are usually neglected because of eddy current dissipation [4]. This approximation is usually assumed in the description of the Barkhausen effect [5], the noise emitted along the hysteresis loop, which indirectly reflects the dynamics of domain walls. The Barkhausen effect is the prototype of the general phenomenon of crackling noise, commonly observed in slowly driven systems with avalanche dynamics [6]. Additional examples are provided by magnetic vortices in type II superconductors [7], ferroelectric materials [8] and driven ionic crystals [9]. In the mechanical context, a notable example is the acoustic emission signal in fracture [10] and plasticity [11] and, on a larger scale, seismic activity in correspondence to an earthquake.

Beside its theoretical interest, quantitative understanding of crackling noise is of fundamental importance in different applications, from non-destructive material testing to hazard prediction. In this context, the average pulse shape has been recently proposed as a sharp tool to characterize crackling noise [6]. In analogy with critical phenomena, it is expected that pulses of different durations can be rescaled into a universal function, which would only depend on general symmetries of the physical process underlying the noise. This is the core of the concept of universality: microscopic details or the sample geometry do not affect

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the scaling behavior. This scenario is supported by the analysis of a variety of models, from disordered spin models to domain wall models, where the pulse shapes are described by universal symmetric scaling functions [12,13]. In virtually all experiments, however, the pulse shape is markedly asymmetric [6,12,14,15]. These results are puzzling because domain wall models describe well several other universal quantities, such as avalanche distributions and power spectra [5,15,16].

Using experimental and theoretical arguments, we have recently came to the conclusion [17] that the asymmetry of the pulse shape is a direct signature of a negative effective mass associated to the domain wall [18]. This negative mass comes from the effect of the dissipative pressure created by the eddy currents during the motion of the domain walls.

In this paper, we aim to explore how the pulse asymmetry is affected by the magnetic domain pattern, by applying an external tensile stress to an high magnetostrictive Fe-base amorphous material. This is particularly important not only because it can be a reliable test for the model, but also because we can better investigate the relevant timescales of the domain wall dynamics. In fact, our calculations and simulations are performed in the approximation of a single domain wall, while in experiments their number is relatively high and this introduces interactions not easily put in a simplified description.

These results help us to clarify the general significance of the pulse shape in crackling noise measurements, and can be very useful for investigating similar features in other systems. The paper is organized as follows: we first present the experimental data performed on an amorphous alloy under different applied tensile stress, showing the resulting analysis of the pulse shape asymmetry and of power spectra. We further present how the effect of the eddy current can be introduced in the calculation of the domain wall dynamics, based on a simple theoretical model, known as ABBM model [19,20]. Using this equation of domain wall motion we simulate the asymmetry values as a function of pulse duration, varying different parameters of the equation. Finally, we briefly discuss the results and the open questions.

2. Experiments

The experiments of Barkhausen noise on soft magnetic materials using a simple inductive setup have been extensively discussed in many papers. We refer the reader to Ref. [5], where details of experimental tricks and features are presented, together with an extensive analysis of the results of the literature. In brief, this technique detects the induced flux using a number of pickup coils wound around a ferromagnetic ribbon, and is suitable for tridimensional samples. In thin films, in contrast, the MOKE technique is usually preferred, even if the experimental detection of the Barkhausen noise and the development of suitable model is still at a quite preliminary stage [21,22].

The present experiments have been performed on a long ribbon (20 cm) of an as cast Fe_{64}Co_{21}B_{15} amorphous alloy, having width of 1 cm, and thickness of 22 μm. The ribbon is subjected to different tensile stress (3, 25, 50, 75 MPa), by applying a weight to one of the ends (from 57 to 1500 g). Usually, measurements of Barkhausen noise are performed at a sampling rate around 20–50 kSamples/s, as a reasonable compromise between temporal resolution and band of amplifiers, which affect the final signal-to-noise ratio. Unfortunately, to detect the average shape we have to increase the sampling rate up to 200 kSamples/s, lowering the signal-to-noise ratio, and requiring a larger statistics. We found that a number of pulses for each stress of the order of 300,000 ensures a reliable estimation of the average shapes.

The pulse are defined and analyzed as follows. We first estimate the distribution of the signal amplitude v, which usually follows a power law with an exponential cutoff $P(v) \sim v^{-1} \exp(-v/v_0)$. Using the value of the cutoff parameter $v_0$, we then introduce a detection threshold (larger than the background noise), in order to define the beginning and the end of a pulse. This threshold is rather small (say, 7–8% of $v_0$) in the case of an high tensile stress because of the high signal-to-noise ratio, while it must be increased up to 10–15% at lower stresses.

As noted in Ref. [23], the property of universality occurring in critical phenomena implies that the signal amplitude should scale following a power law of the type:

$$v(t, T) = T^{1/\alpha_Z} f_{shape}(t/T),$$

where $t$ is the time, $T$ is the pulse duration, and $f_{shape}(t/T)$ is a universal scaling function, those average shape should only depend on general symmetries of the physical process underlying the domain wall dynamics, and not on any microscopic detail. For this reason, $f_{shape}$ has been indicated as a fundamental and stringent tool to test universality. Together with the determination of the critical exponents, these results offer a better comparison of the experimental data with the theoretical predictions, and thus can represent a reliable test for models.

One of the most important critical exponents predicted theoretically and experimentally determined is used in Eq. (1), and correlates the average pulse size $\langle S \rangle$ with its duration $T$ as $\langle S \rangle \sim T^{1/\alpha_Z}$. For this alloy, we have found $1/\alpha_Z \sim 1.62$, independent of the applied tensile stress; this value is slightly smaller than the expected value of 1.77 [5,16]. Using this experimental value in Eq. (1), we have rescaled and averaged the pulses having the same duration $T$ to get the average shape $f_{shape}$ as shown in Fig. 1 for the highest tensile stress (75 MPa, 1500 g). At different stresses the behavior is rather similar, with a nice overall scaling, even if is never perfect. As shown in Fig. 1, many pulses show a marked leftward asymmetry: in particular, shorter pulses show a stronger asymmetry, while longer pulses appear nearly symmetric. This characteristic is strongly enhanced by applying the tensile stress.
To make these observations more quantitative, we need to define a measure of the pulse asymmetry. In Ref. [17] we have proposed to compute the average skewness $\Sigma(T)$ considering

$$\Sigma(T) = \frac{\frac{1}{T} \int_0^T dt (\dot{v}(t, T) (t - \bar{t})^3}{\left[\frac{1}{T} \int_0^T dt (\dot{v}(t, T) (t - \bar{t})^2\right]^{3/2}},$$

where $\bar{t} = 1/T \int_0^T dt (\dot{v}(t, T))t$, and $\langle v(t, T) \rangle$ is the average of $v(t, T)$. With this definition, a positive skewness indicates a leftward asymmetry, that is the signal increases fast and decays more slowly, as in experiments. In general, single pulses do not show any rightward asymmetry, except for rare large pulses which are clearly a superposition of single pulses occurring nearly simultaneously.

In Fig. 2, we show the behavior of the average skewness $\Sigma$ for increasing tensile stress. There are at least two considerations which worth pointing out from this picture: (i) for all stresses, there is a clear range of pulse duration (say $T_p$) where the skewness peaks, up the remarkable value of about 0.27 for the highest stress. These highly asymmetric pulses have a typical duration of $T_p \sim 0.1\text{ ms}$, roughly independent of the tensile stress. These are the shortest pulses shown in Fig. 1; (ii) very short and very long pulses (say, longer than are few milliseconds) have a very low asymmetry, as confirmed by inspection of Fig. 1.

These results are quite surprising for a number of reasons. The Barkhausen effect has been shown to be a true critical phenomena, displaying power laws, scaling etc., and successful models have been based on the depinning critical transition of a single domain wall [5]. All these features do not require, or better, exclude any typical timescale where the dynamical processes occur. The presence of a timescale should thus suggest that universality does not hold anymore, or at least holds only approximately. As a matter of fact, some results reported in the literature have suggested that universality is not extended over all the temporal scales where the Barkhausen signal is detected. In particular, higher order power spectral analysis [24] has showed that scaling sometimes occurs in a limited frequency band, and recognized the presence of a temporal asymmetry well before the average pulse analysis was introduced. A similar, often reported result is the deviation of the power spectrum from the typical $1/f^n$. By the way, these results have never been explained exhaustively.
For these reasons, it thus worth to consider also the power spectrum for this material, as we show in Fig. 3. We can note that a similar deviation from the $1/f$ with $x \sim 1.8$, occurs at a frequency $f^*$ of the order of few kHz, which is fully compatible with the value of $T_p$, as $f^* = (2\pi T_p)^{-1}$. An even more marked deviation around the very same frequency ($\sim 3$ kHz) has been previously reported on a sample with the same composition but subjected to a magnetic annealing under tensile stress [25].

These results allow us to conclude that a new effect, not previously taken into account, acts on the domain wall dynamics on the timescale of the order of $T_p$. To understand the origin of this effect we must first analyze in details the hypothesis usually assumed in the description domain wall dynamics, and investigate where such a dynamical effect can be introduced.

3. Domain wall motion and eddy current dissipation

Traditional approaches to the domain wall dynamics in metallic materials start from some old experimental results showing that the average velocity is proportional to the net field acting on the wall, namely the difference between the applied field $H_{\text{appl}}$ and a coercive field (or depinning field) $H_p$, so that $\langle v \rangle = \mu \omega (H_{\text{appl}} - H_p)$, and $\mu \omega$ is the domain wall mobility. Inertial effects are not relevant up to the GHz band, as the dynamics is fully dominated by eddy current dissipation. This description of the dynamics has been successfully used in the ABBM model to explain many of the scaling properties of the Barkhausen effect [19,20]. In this model, the velocity is assumed at any instant proportional to the net field. This is equivalent to assume that the work done by the net field is instantaneously compensated by the energy dissipation of the eddy currents. If we consider a wall at the position $x$, the equation of motion becomes

$$\beta \dot{x} = 2 I_s (c_H t - k x + W(x)),$$

where $\beta = 1/\mu \omega$ is the damping constant, $I_s$ is the saturation magnetization, $c_H$ is the external field increasing at rate $c_H$, $-k x$ is the demagnetizing field and $W(x)$ is a random field with Gaussian distribution and Brownian correlations [19]. These correlations emerge as a mean-field description of a more general flexible domain wall model [5,26].

Using the solution of the stochastic Eq. (3), it is possible to directly calculate the average shape $\bar{\delta}_\text{shape}$, which turned out to be an arc of sinusoid, a perfectly symmetric function [13,27]. This is not surprising as Eq. (3) turns out to be invariant under time reversal. To include a time asymmetry, we need to investigate the possible dynamical effect of the eddy current dissipation, following the suggestion first pointed out by Bishop, who first considered in detail this effect on the wall motion [18].

3.1. Dynamic effect of eddy current dissipation

Let us consider a slab of cross section $a \times b$ and infinite length, with $a$ along the axis $\hat{x}$, $b$ is the thickness along the axis $\hat{y}$. The main component of the eddy current field produced by the motion of the wall is along the $\hat{z}$ axis, $H_z = H_z(x,y,t)\hat{z}$, and obeys the diffusive equation of the type

$$\sigma \mu \frac{\partial H_z}{\partial t} = \nabla^2 H_z,$$

where $\sigma$ is the conductivity and $\mu$ is the permeability [4]. Eq. (4) must be solved for the slab adding the Faraday condition around the wall

$$\partial_x H_z(0^+,y,t) - \partial_x H_z(0^-,y,t) = 2\pi I_w v(t),$$

where $v \equiv v(t)$ is the domain wall velocity. Eqs. (4) and (5) are solved with the boundary condition $H_z = 0$ at the sample surface. The mean pressure $P_e$ exerted on the wall can then be calculated integrating the field $H_z$ over the thickness, and it is found to be given by a convolution between the velocity $v$ and a response function $f(x)$ as follows:

$$P_e = \frac{2 I_s}{b} \int_{-b/2}^{b/2} H_z(0,y,t) \, dy = \int dt f(t - t') v(t').$$

The effect of eddy currents thus integrates the information of the past velocity of the wall. This is equivalent to say that the wall keeps memory of its past velocity, within a timescale associated to the relaxation times of the response function

$$f(t) = -\frac{32 I_s^2}{\mu \alpha \pi^2} \theta_2 [e^{-t/\tau}] \sum_{n=0}^{\infty} \frac{e^{-t/\tau_n}}{2n + 1} \tau_n,$$

where $\tau_a = \mu \sigma a^2/4\pi^2$, and $\tau_n = \mu \sigma b^2/[(2n + 1)\pi^2]$ are the times related to the sample dimensions $b$ and $a$. It is interesting to compare these times with the time constant introduced in Eq. (3), $\tau_{\text{ABBM}} = \mu a b^2$, where $G \geq 0.1356$ is a geometrical factor. For the sample under investigation, for which $b/a \simeq 10^{-3}$, we have $\tau_n < \cdots < \tau_0 \leq \tau_{\text{ABBM}} \leq \tau_a$. Thus, Eq. (6) introduces a timescale in the domain wall dynamics of the order of $\tau_0$, much smaller of the longest pulses, those duration is typically around $\tau_{\text{ABBM}}$. With this material, we have $\tau_0 = \mu a b^2/\pi^2 \sim 5 \mu$s, $\tau_{\text{ABBM}} \sim 3$ ms, and $\tau_a \sim 8$ s.

We can finally get the equation of motion for the domain wall by equating the eddy current pressure $P_e$ to the pressure $P_a = 2 I_s H$ exerted by the applied field, corrected by demagnetizing effects and pinning, giving

$$\Gamma \ddot{x} + \frac{\Gamma_0}{\tau} \int_0^t \exp(-\tau/\tau_0) \dot{x}(t') \, dt' = 2 I_s (c_H t - k x + W(x)),$$

where $\tau \equiv \tau_0 = \mu a b^2/\pi^2$, and $\Gamma_0$ is a coefficient of the order of $\Gamma$ and $\Gamma + \Gamma_0 = \beta$. It is interesting to reconsider the form of pressure $P_e$ in the frequency domain, $P_e = (\beta + i \omega M^*) \dot{\xi}$: the effect of dynamic eddy currents is thus formally equivalent to a negative effective mass $M^*$ whose value,
in the limit of low frequency $\omega$, is given by

$$M^* \approx \frac{8Ib^3\mu_0^2}{\pi^5} = \frac{\beta \tau}{2}.$$  \hspace{1cm} (9)

This observation enable us to better understand the effect of eddy currents on the domain wall motion: opposite to an inertial drag which limits the initial velocity but tends to keep the velocity during the motion, the eddy currents do not influence the starting velocity but drastically reduce it in a way roughly proportional to its rate of change.

This equivalent mass is a strong function of the sample thickness and frequency dependent, and turns out to be proportional to the damping constant $\beta$ and the relaxation time $\tau$. In this material, we have $M^* \approx -7 \times 10^{-5}$ kg/m$^2$, many orders of magnitude larger than the positive Döring mass ($M^* \approx -10^{-9}$ kg/m$^2$).

4. Simulations

We have simulated the Barkhausen noise using Eq. (8) to compare the skewness and the power spectra with the experimental results. As reported in Fig. 4, the shape of skewness reproduces quite well the experimental results, as it displays a marked peak at about $T_p \approx 10\tau$ and goes to zero at low and high durations. We have performed different simulations varying in a wide range the possible parameters of Eq. (8) (damping constant, demagnetizing field, disorder amplitude, etc.), obtaining values of the maximum no larger than 0.12, which is much smaller than the values obtained in experiments of Fig. 2 at high stress. We will discuss the possible reasons of this discrepancy below.

The simulated power spectrum shows a marked change in the slope at frequency $f^* \approx (2\pi T_p)^{-1}$ (see Fig. 5) in remarkable agreement with the experiments. This enable us to conclude that a marked change in the slope of the power spectrum is a strong indication of the presence of an effective negative mass in the domain wall dynamics. This is an important conclusion as the calculation of the power spectrum is much simpler and faster than the detailed analysis of the pulse shapes required to obtain the skewness.

5. Discussion

Experiments on metallic soft magnetic materials have shown that the dynamic effect related to the eddy current dissipation must be taken into account to describe the asymmetry of the pulse average shape and the detail behavior of power spectra. In this experiment, we have varied the tensile applied stress in order to observe the changes of all these features. The application of an external stress induces a strong uniaxial anisotropy which mainly increases the wall energy: as a consequence, the number of domain walls is reduced and the overall domain pattern simplifies into a series of parallel rigid domains. Despite this different domain structure, the skewness displays a peak for a duration which does not change significantly, at a duration $T_p \approx 20\tau$, which is in reasonably agreement with the value resulting from simulations. It is worth to remark the differences between the equation of motion described by Eq. (8) and the reality of the experiments: while we model a single domain in an effective pinning potential, many domains (of the order several tenths) are present in a sample, with magnetostatic and other interactions difficult to describe with accuracy. In general, we expect, as suggested in Ref. [18], that the interactions between different domain walls could change the effective relaxation time, which in general increases with their number.
In addition, we have to recall that Eq. (3) has been found to describe with a certain accuracy the materials belonging to the so called long-range universality class, where the long-range interactions of magnetostatic origin determine the domain wall dynamics. Amorphous materials under tensile stress belong instead to the short-range universality class, originated by the elastic short-range interaction of the domain wall. We do not know if this is also the reason for the discrepancy between the maximum value of the skewness obtained in simulation and the much larger value found in experiments. At large stresses, we expect to decrease the value of the damping constant (higher mobility), and reduce slightly the overall demagnetizing factor and the amplitude of disorder (a more rigid wall integrates the quenched-in disorder and this is equivalent to a smaller amplitude of fluctuations). Our simulations, however, do not show any significant variation on these parameters. We are currently exploring alternative explanations of this effect, in particular taking into account the universality class to which this material belongs.

On the other hand, we have presented in the past [28] the scaling distributions of duration and size (the area of the pulses) of amorphous materials showing how their scaling exponents are independent of the applied stress, as we have verified again with this sample. This result enforces the idea of universality, and the fact that the interactions dominating the wall dynamics do not change significantly by the application of the stress. But at this point one can significantly pose a general question: does the effective negative mass influence the scaling and universality properties? Or in other words, does the universality still hold or is destroyed by the eddy current dissipation?

Roughly speaking, the effects related to the effective negative mass can be considered as a ‘second order’ perturbation to the domain wall dynamics, i.e. a fine effect limited to a particular range of relaxation times. As a matter of fact, it does not perturb the duration and size distributions, and generally the presence of scaling, with the exception of the deviations shown in Fig. 2. In addition, at long timescales, i.e. for the longest and largest pulse, the asymmetry is recovered and universality holds precisely. Unfortunately, from the experimental point of view, the duration range between $T_p$ and the longest pulses available is rather limited.

These results and the explanation of the pulse asymmetry are very important not only for a better comprehension of the magnetization dynamics in soft magnetic materials, but also to investigate other complex systems showing similar features. Regarding soft magnetic materials, the effective mass encodes important information of the dynamics as it only depends on the damping constant and on the relaxation time $\tau$. We are also currently exploring if and how this effect influences the power losses.

References