Dynamics of a Ferromagnetic Domain Wall and the Barkhausen Effect

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We derive an equation of motion for the dynamics of a ferromagnetic domain wall driven by an external magnetic field through a disordered medium, and we study the associated depinning transition. The long-range dipolar interactions set the upper critical dimension to be $d_c = 3$, so we suggest that mean-field exponents describe the Barkhausen effect for three-dimensional soft ferromagnetic materials.

Here, we present an accurate treatment of magnetic interactions in the context of the depinning transition, which allows us to explain the experiments and to give a microscopic justification for the model of Ref. [13]. We study a single domain wall separating two regions with magnetization of constant magnitude $M$ and opposite directions. We assume that the wall surface does not form overhangs, and describe the position of the wall by its height $h(\tilde{r},t)$. The motion of the wall is overdamped because of eddy currents so, neglecting thermal fluctuations, the evolution of $h(\tilde{r},t)$ is governed by

$$ \frac{\partial h(\tilde{r},t)}{\partial t} = -\delta E([h(\tilde{r},t)]) \delta h(\tilde{r},t), \quad (1) $$

where $E([h(\tilde{r},t)])$ is the energy of the system [14,15]. We will show that by incorporating the effects of ferromagnetic, magnetocrystalline, magnetostatic and dipolar interactions, and disorder, the equation of motion is

$$ \frac{\partial h(\tilde{r},t)}{\partial t} = v_0 \nabla^2 h(\tilde{r},t) + H - H_d + \int d^2 r' K(\tilde{r} - \tilde{r}') \times [h(\tilde{r}',t) - h(\tilde{r},t)] + \eta(\tilde{r},t). \quad (2) $$

Here $v_0$ is the surface tension, $H$ is the external magnetic field, the demagnetizing field $H_d$ and the nonlocal term are due to dipolar interactions, and we model the disorder with a random force, with Gaussian distribution and short range correlations,

$$ \langle \eta(\tilde{r},h)\eta(\tilde{r}',h') \rangle = \delta^2(\tilde{r} - \tilde{r}')\Delta(h - h'), \quad (3) $$

where $\Delta(x)$ decays very rapidly for large values of the argument.

The dipolar interactions are treated considering effective magnetic charges induced by the discontinuities of the magnetization across the boundaries of the sample and the domain wall [16]. The corresponding magnetic surface charge is given by

$$ \sigma = (\mathbf{M}_1 - \mathbf{M}_2) \cdot \hat{n}, \quad (4) $$
where \( \hat{n} \) is normal to the surface and \( \vec{M}_1 \) and \( \vec{M}_2 \) are the magnetization vectors on each side of the surface. The surface charge induced at the boundary of the sample gives rise to a demagnetizing field which opposes the external field [12]. The simplest approximation is to consider this field to be constant throughout the sample, and to be proportional to the total magnetization [9,13]. This yields the term \( H_d = -k \int d^2r \hat{n}(\vec{r},t) \) in Eq. (2), where the demagnetizing factor \( k \) takes into account the geometry of the domain structure, the shape of the system, and its size.

Magnetic surface charges will also appear on the domain wall when its surface is not parallel to the magnetization. In the limit of infinite anisotropy [17] and small bending of the surface, we can express the surface charge as

\[
\sigma(\vec{r}) = 2M \cos \theta \approx 2M \frac{\partial h(\vec{r},t)}{\partial x},
\]

where \( \theta \) is the local angle between the vector normal to the surface and the magnetization (see Fig. 1). The interaction (6) is long range and anisotropic, as can be seen by considering the Fourier transform of \( \sigma(\vec{r}) \) in Eq. (2), where \( \hat{h} \) is normal to the surface and \( \vec{r} \) yields the term

\[
K(\vec{r} - \vec{r}') = \frac{2M^2}{|\vec{r} - \vec{r}'|^3} \left( 1 + \frac{3(x - x')^2}{|\vec{r} - \vec{r}'|^2} \right).
\]

The interaction (6) is long range and anisotropic, as can also be seen by considering the Fourier transform

\[
K(p,q) = \frac{2M^2}{\pi} \frac{p^2}{\sqrt{p^2 + q^2}},
\]

where \( p \) and \( q \) are the two components of the Fourier vector. Moreover, an estimate of the order of magnitude of \( K(\vec{r} - \vec{r}') \) shows that it dominates over the surface tension for all length scales of interest [19].

Apart from the nonlocal kernel, Eq. (2) is the equation proposed in Ref. [9] which, in turn, reduces when \( k = 0 \) to a driven elastic interface in the presence of quenched disorder [20–22].

For \( k = 0 \) (no demagnetizing field), Eq. (2) displays a depinning transition; i.e., there exists a critical field \( H_c \) such that for \( H < H_c \) the interface is pinned while for \( H > H_c \), it moves with nonzero average velocity. At \( H = H_c \), the system exhibits scaling properties: The interface moves by avalanches whose sizes \( s \) and durations \( T \) distributions follow power laws,

\[
P(s) \sim s^{-\tau}, \quad P(T) \sim T^{-\alpha}.
\]

When \( k > 0 \), the demagnetizing field provides an additional restoring force that keeps the interface at the depinning transition [9,11] if the external field is increased adiabatically.

Using the functional renormalization group scheme introduced in Ref. [23], we find that—due to the long-range kernel \( K(\vec{r} - \vec{r}') \) in Eq. (6)—the critical behavior of Eq. (2) differs from that of elastic interfaces: The upper critical dimension becomes \( d_c = 3 \) [18,24,25], instead of \( d_c = 5 \) [21,22]. Hence we predict that, for \( d = 3 \), the motion of the domain wall will be described by mean-field theory (apart from logarithmic corrections), which yields \( f_{\text{up}} = 0.7 \) [22,26].

Next we make contact between our approach and the conventional approach that reduces the domain wall to a single point moving in a random pinning field [3,13,27]. To this end, we introduce an infinite range version \( (d \to \infty) \) of Eq. (2), which should have the same critical behavior as Eq. (2) but has the advantage of being much simpler to analyze.

To treat the infinite-range model, we discretize the interface and consider that all \( N \) elements are at the same distance from each other. Equation (2) then becomes [28]

\[
\frac{\partial h_i(t)}{\partial t} = H(t) - \dot{\chi} \hat{h}_i + J[\hat{h} - h_i(t)] + \eta_i(h),
\]

where \( \hat{h} = \sum_{i=1}^{N} h_i/N, \chi = N\chi, J = (\nu_0 + 2M^2) \), and the external field \( H(t) \) increases at a finite constant rate. Summing Eq. (9) over all sites \( i \), we obtain an equation for the total magnetization \( m = Nh \),

\[
\frac{dm}{dt} = \dot{\chi} m + \sum_{i=1}^{N} \eta_i(h),
\]

where the time dependence of the field has been made explicit. We can approximate \( \sum \eta_i \) by an effective random pinning field \( W(m) \), depending only on the magnetization. When the interface moves between two configurations, the change in \( W \) is

\[
W(m') - W(m) = \sum_{i} \Delta \eta_i,
\]

where the sum is restricted to the sites that have moved (i.e., their disorder is changed). The total number of such
sites scales, on average, as $|m' - m|$, since for $d \approx d_c$ the size of an avalanche scales like its area [18,22]. Assuming that the $\Delta \eta_i$ are uncorrelated and have random signs, we find that the effective pinning field is correlated,

$$\langle |W(m') - W(m)|^2 \rangle = D|m' - m|,$$

where $D$ sets the scale of the fluctuations of $W$. These correlations of “Brownian” type have been experimentally observed in SiFe alloys [3]. In the model of Alessandro et al. [13] (ABBM), the domain wall is treated as a single point moving in a Brownian correlated field. The ABBM model is equivalent to Eqs. (10) and (12), and predicts that the avalanche exponents should depend on the field driving the system. In Ref. [9] (i.e., $\tau \approx 1.3$) could be due to a finite driving rate ($c = 0.4$). In addition, it is important to remark that the present theory applies only if domain wall motion is the dominant magnetization process. This was indeed the case in Refs. [4,5,29], where the noise was recorded only in the region of constant permeability around the coercive field. A detailed critical discussion of the experimental results reported in the literature can be found in Ref. [29].

In particular geometries, typically frames or toroid samples, the demagnetizing field is absent ($\chi = 0$) [3]. It is then possible to observe experimentally the depinning transition of domain walls. It has been reported that the average velocity of the domain walls, in different ferromagnetic materials [30], increases for $H > H_c$, as

$$v \sim (H - H_c)^{\beta},$$

with $\beta = 1$, in agreement with the theory ($\beta = 1$ is expected in mean-field theory [22]).

We have seen that avalanche distributions can be described by power laws with exponents that do not depend on material details. The power spectrum of the noise displays instead a more complex structure and does not show such a marked universality. At low frequency the power spectrum grows with an exponent varying between $\psi = 0.5$ for crystalline alloys and $\psi = 1$ for amorphous...
alloys, while at high frequencies it decays with an exponent varying between $\psi = -2$ for crystals and $\psi = -1.6$ for amorphous alloys [2, 4–6, 29]. In samples with a single domain wall present, the power spectrum was found to decay as $\omega^{-2}$ [3].

Following the analysis of Tang et al. [31], we obtain, at low frequency, $\psi = 1$ for the power spectrum measured on a single site and $\psi = 0$ when the signal is averaged over the whole system. For the averaged spectrum, we also find a $\omega^{-2}$ decay at large frequencies, due to the Brownian properties of the effective pinning field. The discrepancies between theory and experiments could be due to the presence of many domain walls interacting through the demagnetizing field. When a domain wall starts to move, the demagnetizing field increases, creating a larger pinning force on the other walls. Therefore, on short time scales the interactions between the walls is irrelevant and should not change the avalanche distributions. On larger time scales, this effect may be important and could modify the properties of the power spectrum. In order to clarify this issue, it would be necessary to analyze in detail the dynamics of many coupled domain walls.

The present theory for the Barkhausen effect, based on the depinning of a ferromagnetic domain wall, should apply to soft ferromagnetic materials, which are frequently used in experimental studies of the Barkhausen effect [2–6, 9, 13]. For hard ferromagnets and rare earth materials where strong random anisotropies prevent the formation of domains, disordered spin models could be appropriate [8].

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[17] In the case of finite anisotropy, the direction of the magnetization can deviate from the easy axis. This effect can be included in the derivation, and the resulting kernel is qualitatively unchanged [14].
[19] The long-range kernel becomes relevant for length scales larger than $L \sim v_0/M^2$. In typical ferromagnets, $M \sim 10^3$ and $v_0 \sim 1$ (in cgs units) (see page 713 of Ref. [12]). This implies $L \sim 10^{-6}$ cm, which is of the order of the domain wall thickness.