Dipolar interactions in ferromagnetic systems: Dynamic hysteresis from parallel domain walls

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Abstract

We present a model for the study of the hysteretic behavior of a disordered ferromagnetic system with an array of parallel Bloch domain walls. We write the equations of motion of the walls under an external magnetic field driving, considering long-range dipolar interactions and disorder. We calculate analytically an expression for the magnetic susceptibility \( \chi \), and find a logarithmic dependence for \( \chi \) on the number of domains. We study the dynamic hysteresis, which behavior could be explained by the theory of loss separation.

1. Introduction

The study of ferromagnetic hysteresis represents an open field of current interest, due to its wide technological applications [1]. Theoretically speaking, one of the central issue in ferromagnetic systems physics is the link between the domain structure and the hysteretic properties. The dependence of the hysteresis loop area on the frequency and the amplitude of the applied magnetic field is called dynamic hysteresis [2,3] in thin films context and power losses in bulk materials. In the latter case, the energy dissipation is dominated by eddy currents [1], while in two dimensions this effect is negligible.

Nevertheless, it was recently shown that the theory of loss separation could be applied to thin films, whatever the damping nature [4].

In this contribution, we consider a system with rigid parallel Bloch domain walls, a common configuration both for bulk and two-dimensional ferromagnets. We introduce a model for the motion of the domain walls in a disordered material, driven by an external magnetic field. The model is based on the interplay between the dipolar and the external field contributions, in the presence of structural disorder.

Due to the simplicity of the walls configuration, we can calculate perturbatively the magnetic susceptibility \( \chi \) as a function of the structural parameters of the system, and find agreement with the simulations results. We can thus link a macroscopic measurable quantity and the microscopic dynamics of the system. We analyze the dynamic hysteresis by integrating numerically the equations of motion of the domain walls, and find a good agreement with the theory of loss separation.

2. The model

We consider an array of Bloch domain walls in the position \( \mathbf{x}(t) = (x_0(t), x_1(t), x_2(t), \ldots, x_n(t)) \), at time \( t \). As we are interested in the macroscopic response, we do not consider the details of the internal structure of the walls, and treat only the magnetostatic, the disorder and the external field contributions. Thus the total force \( F(k, t) \) acting on the \( k \)th wall at time \( t \) is given by

\[
F(k, t) = F_m(k, t) + F_{\text{dis}}(k, t) + F_{\text{ext}}(k, t).
\]
The magnetostatic term $F_m(k,t)$ takes into account the interaction between the magnetization and the stray field generated by the magnetic charges due to discontinuity of the magnetization vector at the upper and lower boundaries of the sample, and it is derived from the Maxwell’s equations. It results:

$$F_m(k,t) = 2M_s^2\mu_0 \sum_{j=0}^{n-1} (-1)^{j+k} \left[ \frac{\partial g(x_{j+1}, x_k)}{\partial x_k} - \frac{\partial g(x_j, x_k)}{\partial x_k} \right] - \varepsilon^2 \left( \frac{\partial f(x_{j+1}, x_k)}{\partial x_k} - \frac{\partial f(x_j, x_k)}{\partial x_k} \right)$$

(2)

being $M_s$ is the saturation magnetization of the material and $\mu_0$ the vacuum permeability, and

$$\frac{\partial f(x,y)}{\partial y} = - \frac{\partial f(x,y)}{\partial x} = \ln \left( \frac{y - x + \sqrt{4d^2 + (x - y)^2}}{x - y + \sqrt{4d^2 + (x - y)^2}} \right),$$

$$\frac{\partial g(x,y)}{\partial y} = - \frac{\partial g(x,y)}{\partial x} = 2(x - y) \sqrt{\varepsilon^2 + (x - y)^2}$$

$$+ 2\varepsilon(x - y) \ln \left( \frac{\sqrt{\varepsilon^2 + (x - y)^2} - \varepsilon}{\sqrt{\varepsilon^2 + (x - y)^2} + \varepsilon} \right)$$

$$+ \varepsilon^2 \ln \left( \frac{y - x + \sqrt{\varepsilon^2 + (x - y)^2}}{x - y + \sqrt{\varepsilon^2 + (x - y)^2}} \right)$$

$$- 2(x - y)^2 \text{sign}(x - y).$$

(3)

$F_{\text{dis}}(k,t)$ models the contribution of structural disorder, impurities, defects and is due to the sum of randomly placed pinning centers each interacting with the wall by a Gaussian potential:

$$U_p(x) = A \exp[-(x/\xi)^2],$$

(4)

where $x$ is the distance between the pinning center and the wall, and the amplitude $A$ is extracted from a uniform distribution, considering that the strength of the pinning centers should vary for structural reasons, and $\xi$ is an interaction range of the order of the domain amplitude (Fig. 1).

$F_{\text{ext}}(k,t)$ represents the interaction between the magnetization and the external magnetic field. In formula:

$$F_{\text{ext}}(k,t) = 4\mu_0 M_s d \varepsilon H_{\text{ext}}(t)(-1)^{k+1}.$$  

(5)

Model details will be described elsewhere [5].

3. The magnetic susceptibility

Starting from the equilibrium configuration, we calculate the magnetic susceptibility $\chi$ as the magnetization variation of the system under a perturbative external field, i.e.

$$\chi = \frac{dM}{dH_{\text{ext}}}.$$  

For a large and even number of domains the equilibrium configuration is periodic, $x_i = iL/n \forall i$. We can develop the total force acting on a generic wall close to the equilibrium. Due to the simplicity of the wall configuration, these calculations could be handled analytically and lead to (see Ref. [5])

$$\chi = \frac{d}{\varepsilon A(n)}$$

(6)

being $2d$ is the sample height, $\varepsilon$ the thickness, $n$ the number of domains and $A(n) = 2[0.577215 + \ln(n)]$. In Fig. 2 we compare the analytic result with the simulations. The agreement is very good, and the discrepancies are due to finite system size effects in the simulations.

4. Simulations: dynamic hysteresis

We write the equations of motion of the walls as overdamped equations:

$$\Gamma \frac{dx_k}{dt} = F_m(k,t) + F_{\text{ext}}(k,t) + F_{\text{dis}}(k,t).$$

(7)

![Fig. 1. Sketch of the parameters of the system, for an array of walls positions $\{x_1, x_2, \ldots, x_n\}$.](image1)

![Fig. 2. Magnetic susceptibility $\chi$ as a function of the system length $d$: comparison between perturbative calculations and simulation results for $n = 70$ and $\varepsilon = 0.002$ and total width $L = 1$.](image2)
where we set $\Gamma = 1$, and integrate them by a fourth order Runge–Kutta algorithm, using fixed boundary conditions (for the details see Ref. [5]).

We study the dynamic hysteresis by means of the behavior of the coercive field $H_c$, see Fig. 3. As it can be seen, the data are very well fitted by the loss separation formula [4],

$$H_c = C_{ex}[1 + r\dot{H}]^{1/2} - 1,$$

where $C_{ex} = n_0 V_0 / 2$, $r = 4\gamma \mu / (n_0^2 V_0)$, $n_0$ is the number of active walls in the quasistatic limit, $V_0$ is a characteristic field which controls the increase of $n_0$ due to the excess field, and $\mu$ is the permeability.

5. Conclusions

We have developed a model for the motion of an array of parallel Bloch domain walls, considering the dipolar interactions, the external field and the disorder. We have calculated analytically the perturbative magnetic susceptibility, that results in good agreement with the simulations. Moreover, we have studied the dynamic hysteresis by means of the coercive field behavior, and find that its behavior is very well fitted by the theory of loss separation.

References